Introduction to Modern Cosmology Spring 2020

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1. State the units (the **dimension**) in the cgs system of the following quantities:

- D, the size of the Solar system.
- G, Newton's constant.
- ω, orbital angular velocity of Mercury
- p, the momentum of a comet.

2. Lemaitre explained Slipher's nebulae (1917) redshifts out to about $z \sim 0.2$ in terms of an expanding Universe, which marked the beginning of modern astronomy. A first systematic approach to measuring the Hubble parameter, H_0, was explored by Hubble. Hubble's advance was made possible by

- A larger telescope, seeing out further out than Slipher.
- A systematic search for Cepheid variables in distant nebulae permitting use of Leavitt's law to extract distances.
- A systematic search for distant nebulae, extracting distances from their apparent surface brightness.
- The theory of general relativity, giving a model for the expansion of the Universe.

3. Minkowski spacetime provides a suitable starting point to model particle interactions. For reference, it suggests using a total mass-energy $E = Mc^2$ at infinity. In cosmological spacetime, however, we face a cosmological horizon at a finite Hubble radius $R_H = c/H$, where c is the velocity of light and H is the Hubble parameter. In light of this, we consider, instead, mass-energy densities ρc^2 normalized to closure density $\rho_c = 3H^2/8\pi G$, where G is Newton's constant.

- If today $\rho > \rho_c$, the Universe will expand forever.
- If today $\rho > \rho_c$, we had $\rho < \rho_c$ some time in the past.
- If today $\rho \leq \rho_c$, the Universe will expand forever.
- If today $\rho = \rho_c$, the Universe has no beginning or end.

4. Consider galaxy surveys with modest depth (maximal observable redshift) in early attempts to measure the Hubble parameter H_0 in the Hubble-Lemaître law $v = H_0 D$, where (v, D) refers to the receding velocity v of a galaxy at distance D. One source of scatter is the peculiar velocity v_p of individual galaxies. Recall that, very roughly, typical distances between galaxies are on the order of 1Mpc. In the Local Universe, we have $D = zR_H$ scaled to the Hubble radius $R_H = c/H_0$ $(D << R_H)$, where c is the velocity of light.

- For a Hubble parameter $H_0 \simeq 73\mathrm{km\,s^{-1}Mpc^{-1}}$, derive R_H in Gpc.
- Derive a scale of v_p, that may appear naturally in galaxy clusters.
- Derive a minimum redshift depth for a survey to measure H_0 with no need to correct for v_p.

5. Galaxy evolution is strongly affected by, e.g., supernova history. Let $\beta_e = v_e/c$ denote the escape velocity v_e normalized to the velocity of light, c. According to Newton's law, $\beta_e^2 = R_g/r$ at a distance r of a mass M with gravitational radius $R_g = GM/c^2$. Recall the solar radius $R_\odot \simeq 700,000\,\mathrm{km}$. For a typical Milky Way type galaxy, the central bulge is of size $R \simeq 5\,\mathrm{kpc}$.

- Argue that β_e is very similar for stars and the host galaxy.
- Explain that supernova history is an important factor in galaxy evolution *overall*.
- State three other factors governing galaxy evolution over cosmological time scales.