Spring 2020

Introduction to Modern Cosmology

Maurice H.P.M. van Putten

Department of Physics and Astronomy

MO and WE

Online / March 2020

(c)2020 van Putten

Overview of this course

- Modern astronomy: basic concepts
- How large is the Universe?
- **The evolving Universe: Hubble parameter**
- Early universe following the Big Bang
- Present-day cosmology
- Key open questions

(c)2020 van Putten

Contents - Week 3

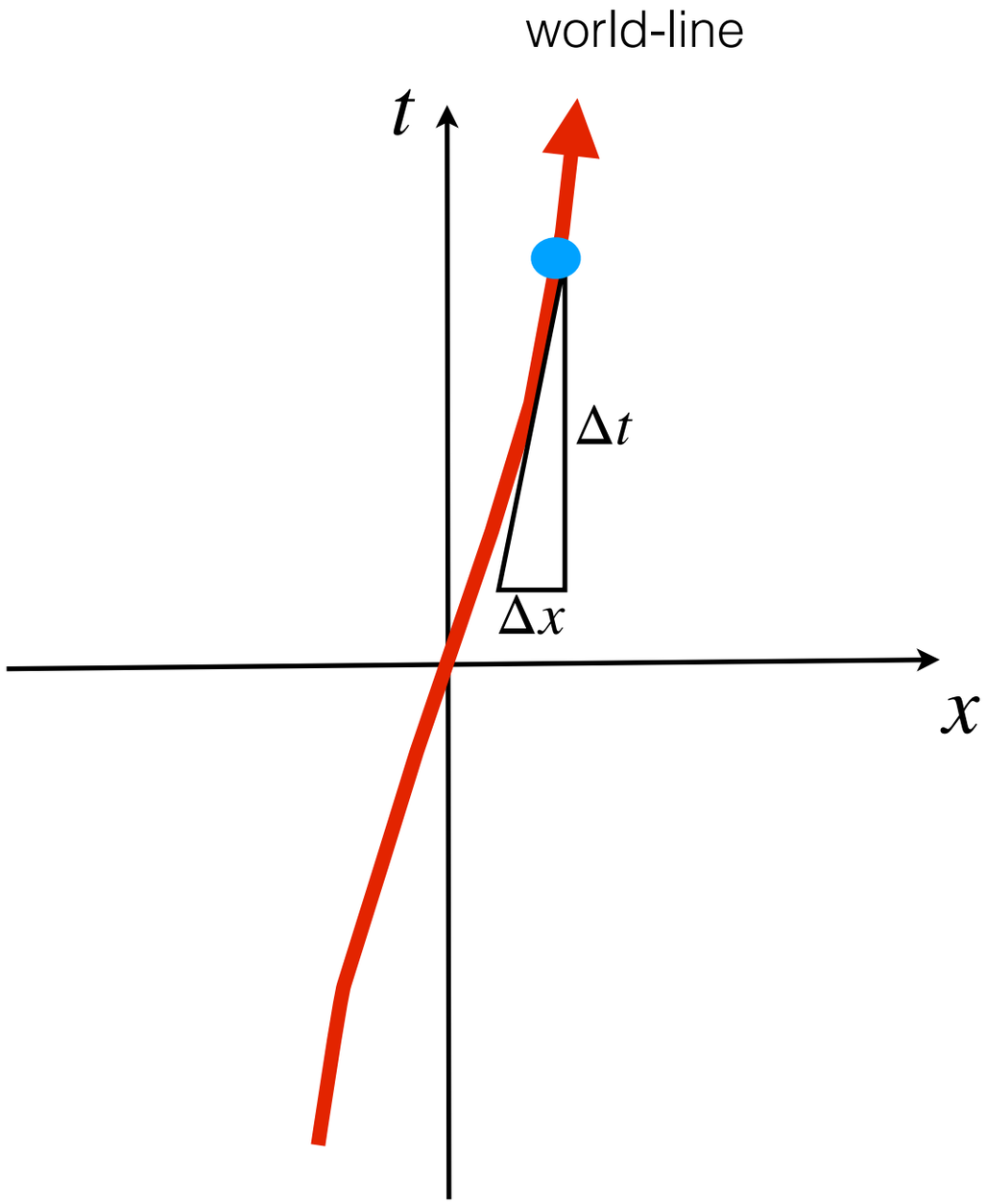
W3.1 Relativistic mechanics

W3.2 Lorentz transformation

W3.3 Some special relativistic effects

(c)2020 van Putten

Particle world-lines



World-line: particle trajectory in space and time

Three velocity defined by the ratio of intervals of space and time:

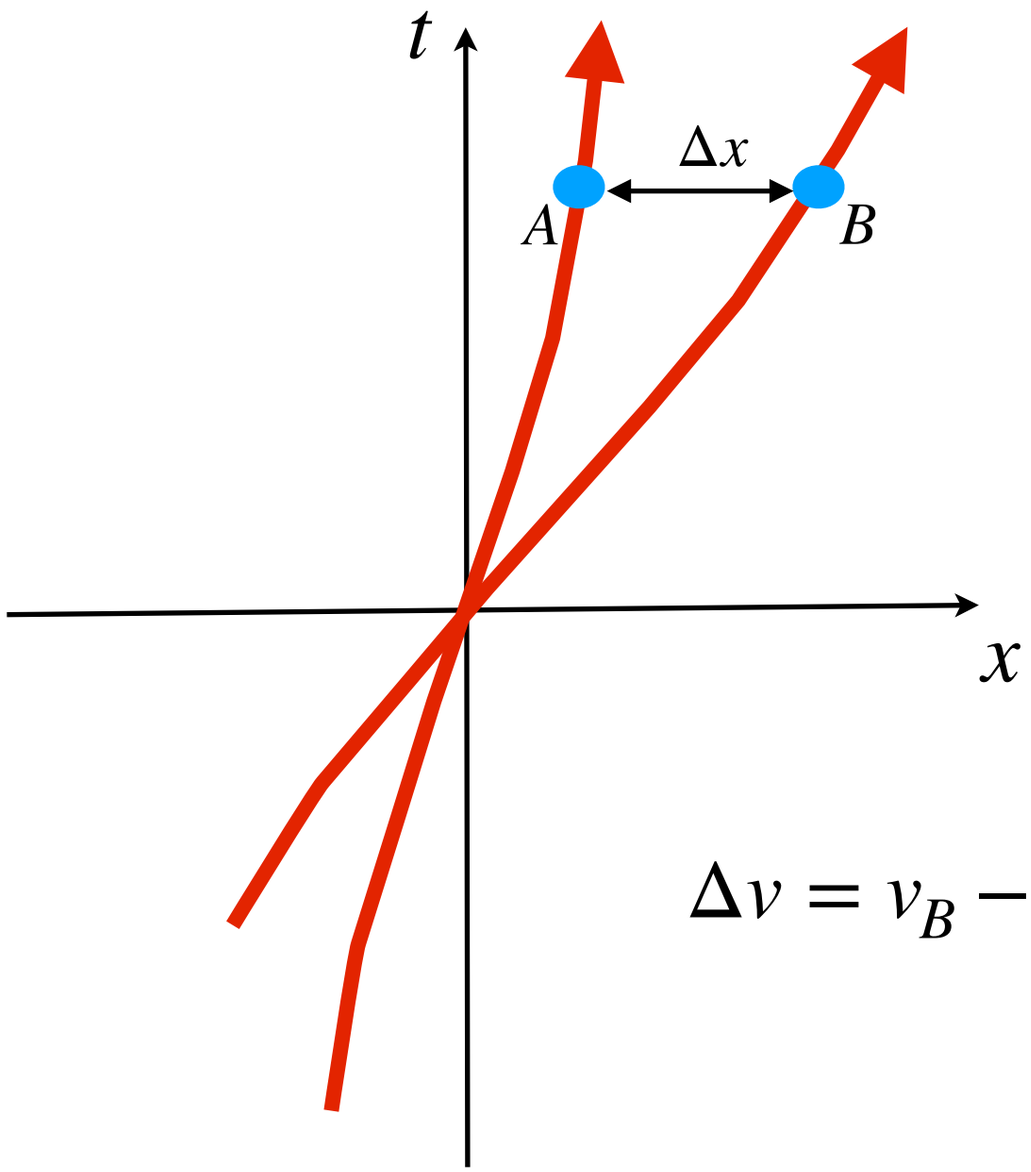
$$v = \frac{\Delta x}{\Delta t}$$

1864-1909

Newtonian-Galilean spacetime: everyone agrees on the same time-interval

(c)2020 van Putten

Relative velocity



Galilean relativity:

$$v_A = \frac{\Delta x_A}{\Delta t}, \quad v_B = \frac{\Delta x_B}{\Delta t}$$

(Displacements relative to the origin:
$\Delta x_A = x_A,\ \Delta x_B = x_B$)

$$\Delta v = v_B - v_A = \frac{\Delta x_B - \Delta x_A}{\Delta t} = \frac{x_B - x_A}{\Delta t} \equiv \frac{\Delta x}{\Delta t}$$

Everyone agrees also on the same separation *AB*, whence their relative velocity is absolute

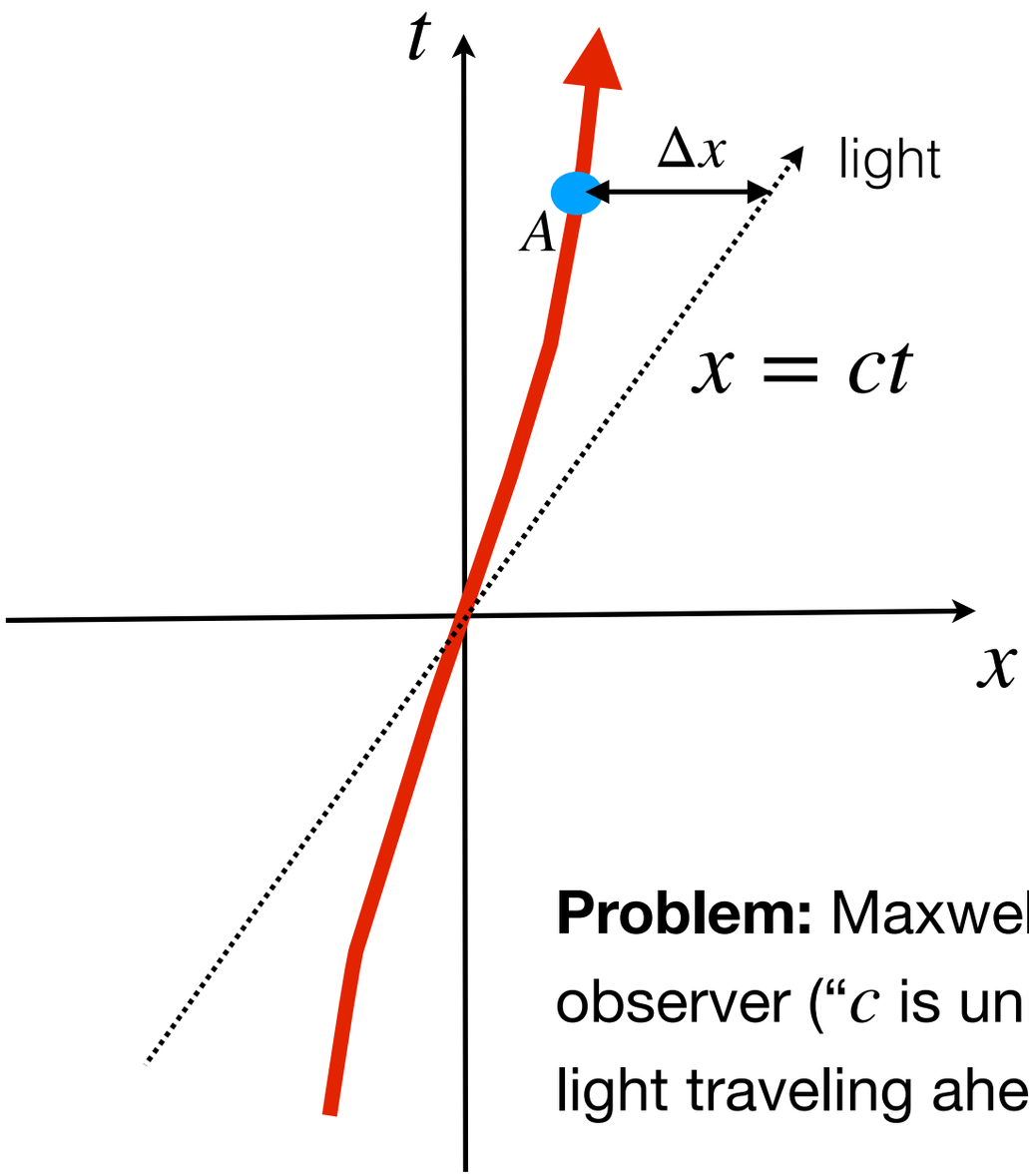
(c)2020 van Putten

What is time, light?

https://www.youtube.com/watch?v=ZyYqyYAKGC0

Chasing light…

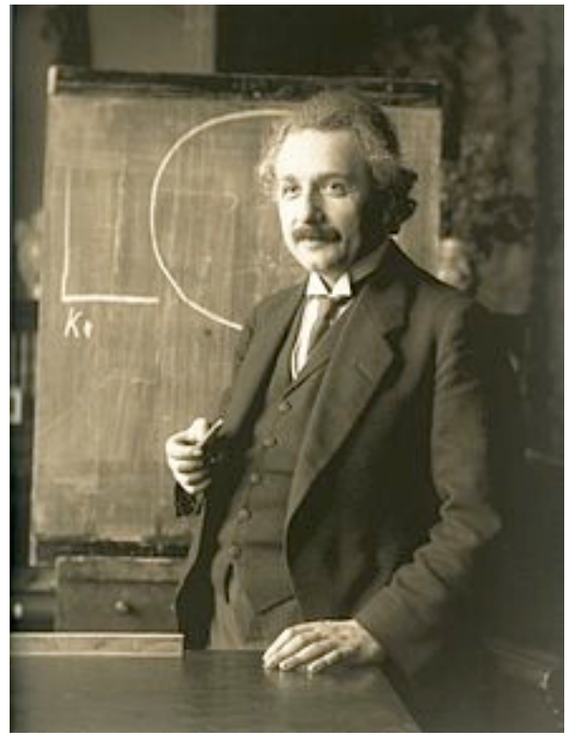
An observer at rest in the laboratory frame sees *A* trailing light:



This laboratory observer infers a relative velocity

$$\Delta v = c - v_A < c$$

By Galilean relativity, Δv is absolute, whence *A* must see light traveling ahead away with $\Delta v < c$.



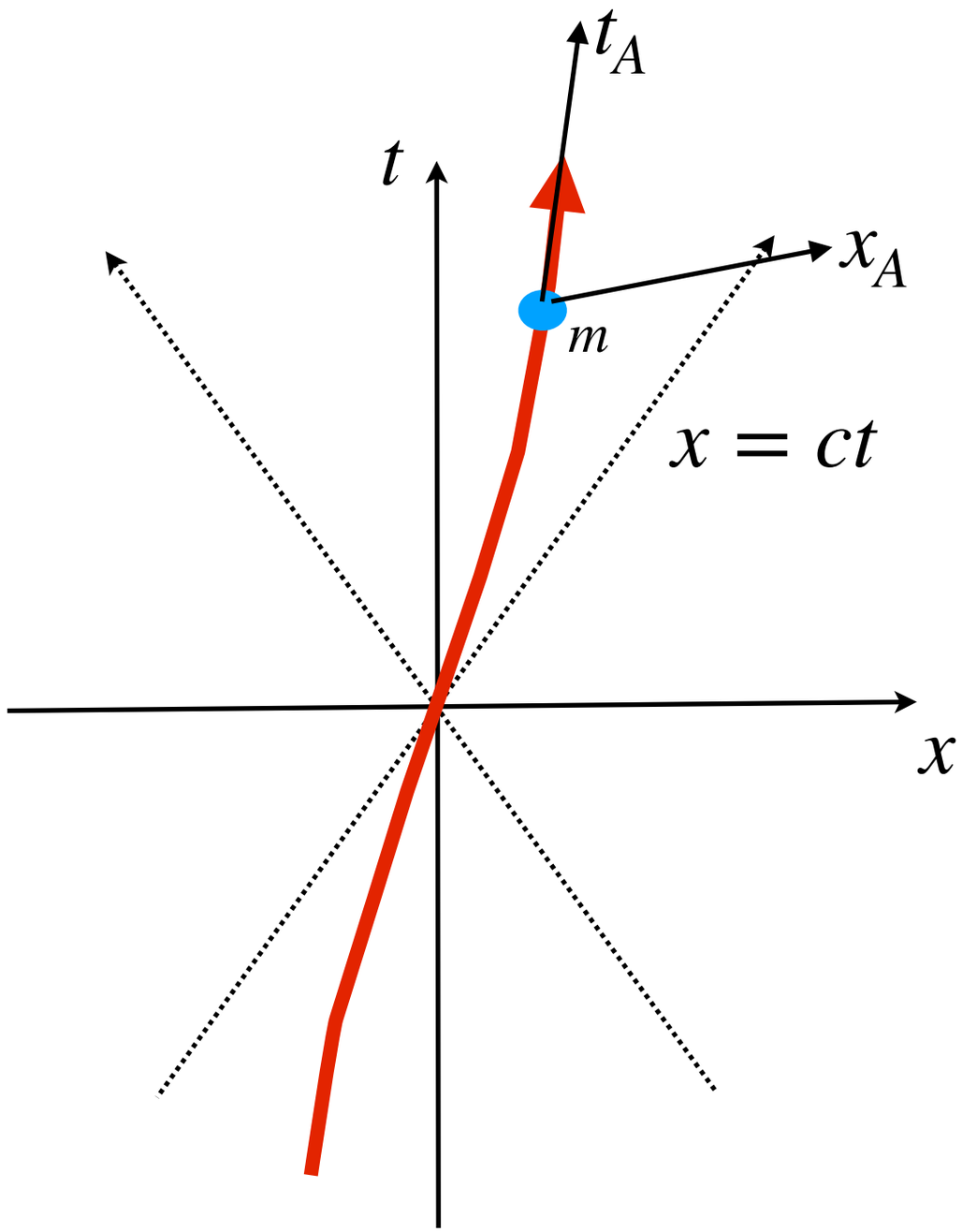
A. Einstein 1879-1955

Problem: Maxwell's equations show c is the same for every (inertial) observer ("c is universal"). In its coming frame of reference, *A* sees light traveling ahead also at velocity c.

Δv cannot be absolute: space and time intervals cannot be absolute

(c)2020 van Putten

Minkowski spacetime



Comoving coordinate system of A:

$$(t, x)_A = (t_A, x_A)$$

Allow for non-trivial transformations of space and time intervals

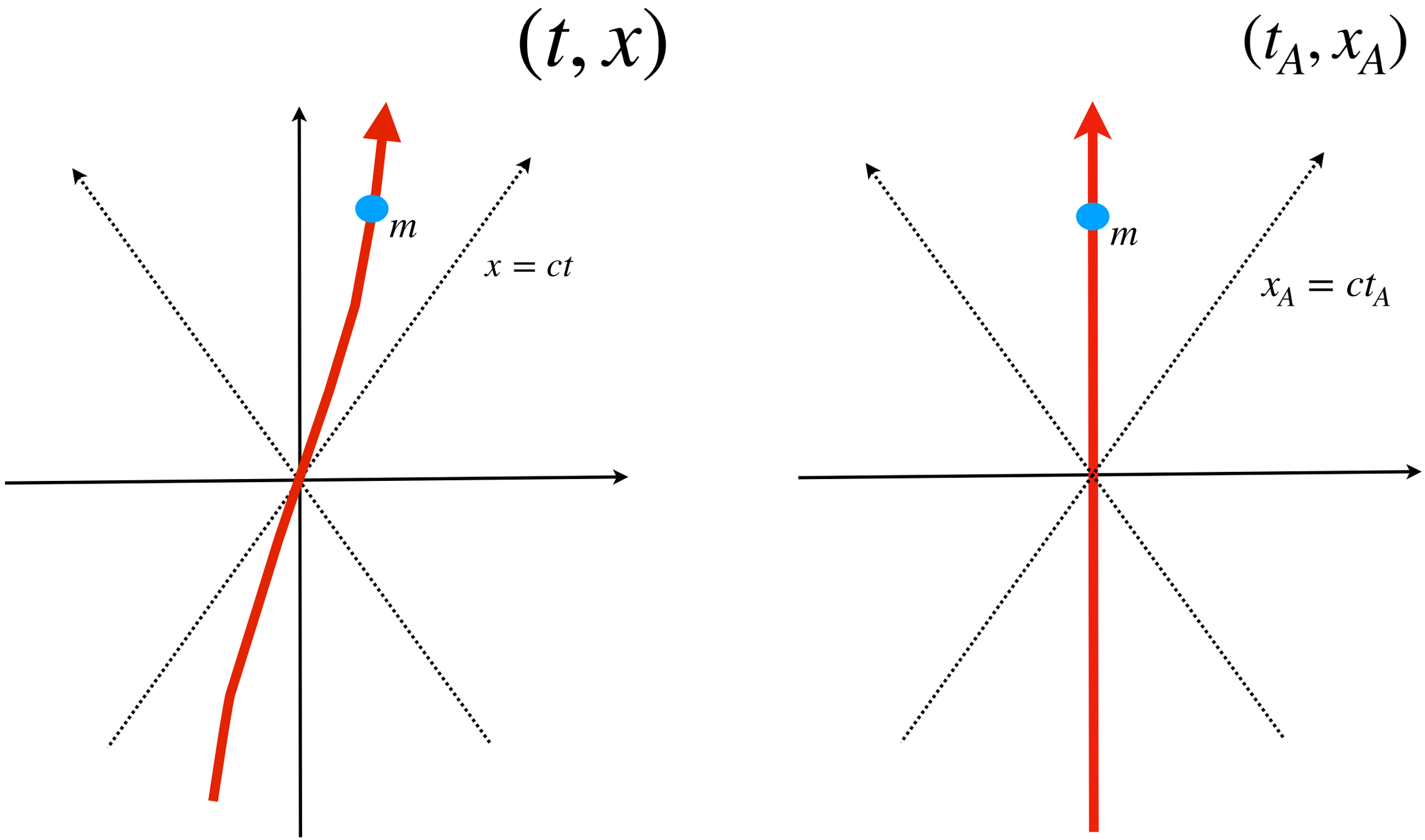
$$\Delta t_A = f_\lambda(\Delta t, \Delta x)$$

$$\Delta x_A = g_\lambda(\Delta t, \Delta x)$$

At any velocity of *A*, *A* will see the same velocity *c* of a photon, measured in its coming frame of reference:

$$\left(\frac{\Delta x}{\Delta t}\right)_A \equiv c$$

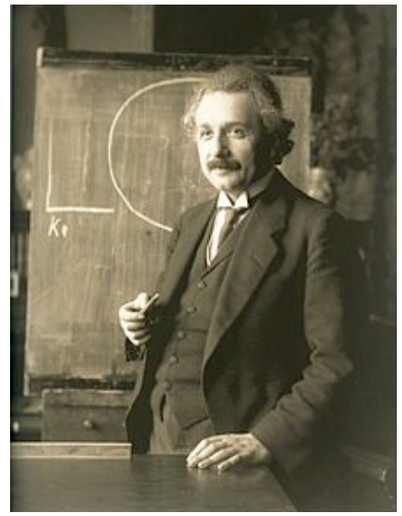
Invariance of light cones



Laboratory observer and observer *A* both see the same light cone as defined by c in their respective frames of reference

(c)2020 van Putten

Invariant distance measure

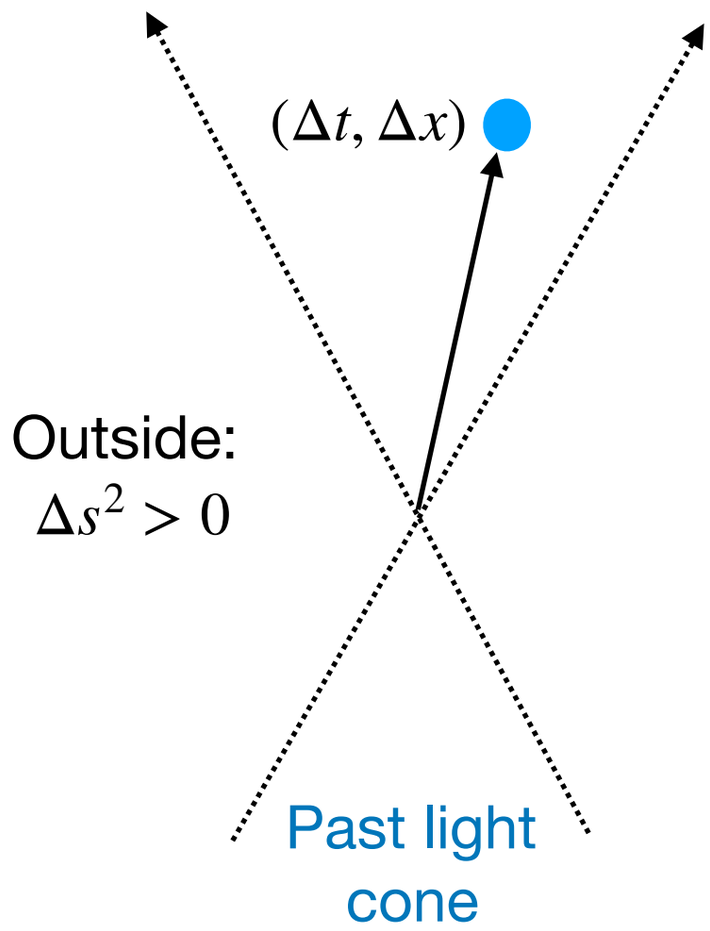

A. Einstein 1879-1955

Consider the signed measure

$$\Delta s^2 = -c^2\Delta t^2 + \Delta x^2$$

with light cone

$$\Delta s^2 = 0.$$



The inside and outside of a light cone are invariant regions:

Sign of Δs^2 is observer invariant

- Einstein: Δs^2 itself is observer invariant
- Invariant signed measure $ds^2 = -c^2dt^2 + dx^2$

Minkowski spacetime

1864-1909

- Minkowski line-element: $ds^2 = -c^2dt^2 + dx^2$
- Eigentime $\tau = t_A$ in the comoving frame of an observer A
- Invariance of the signed measure ds: $ds^2 = -c^2dt^2 + dx^2 = -c^2d\tau^2$

In the laboratory frame of reference, the coordinates of *A* satisfy the normalization

$$\left(\frac{dt}{d\tau}\right)^2 - \left(\frac{dx}{cd\tau}\right)^2 = 1$$

Velocity four-velocity

Put $c = 1$, $d\tau = ds$ and consider the four-velocity

$$x^b = (t, x): \quad u^b = \left(\frac{dt}{ds}, \frac{dx}{cds} \right)$$

In Minkowski spacetime, we arrive at a normalization on u^b :

$$u^t u^t - u^x u^x = 1$$

It has one free parameter representing rotation over an imaginary angle:

$$e^{i\alpha} = \cos\alpha + i \sin\alpha$$

$$\cos\alpha = \frac{e^{i\alpha} + e^{-i\alpha}}{2} = \mathrm{Re}\, e^{i\alpha}$$

$$\sin\alpha = \frac{e^{i\alpha} - e^{-i\alpha}}{2i} = \mathrm{Im}\, e^{i\alpha}$$

$$\cos^2\alpha + \sin^2\alpha = 1$$

$$e^{\lambda} = \cosh\lambda + \sinh\lambda$$

$$\cosh\lambda = \frac{e^{\lambda} + e^{-\lambda}}{2} = \cos i\lambda$$

$$\sinh\lambda = \frac{e^{\lambda} - e^{-\lambda}}{2} = -i \sin i\lambda$$

$$\cosh^2\lambda - \sinh^2\lambda = 1$$

(c)2020 van Putten

Four- and three-velocity

Consider the four-velocity in terms of the rapidity λ :

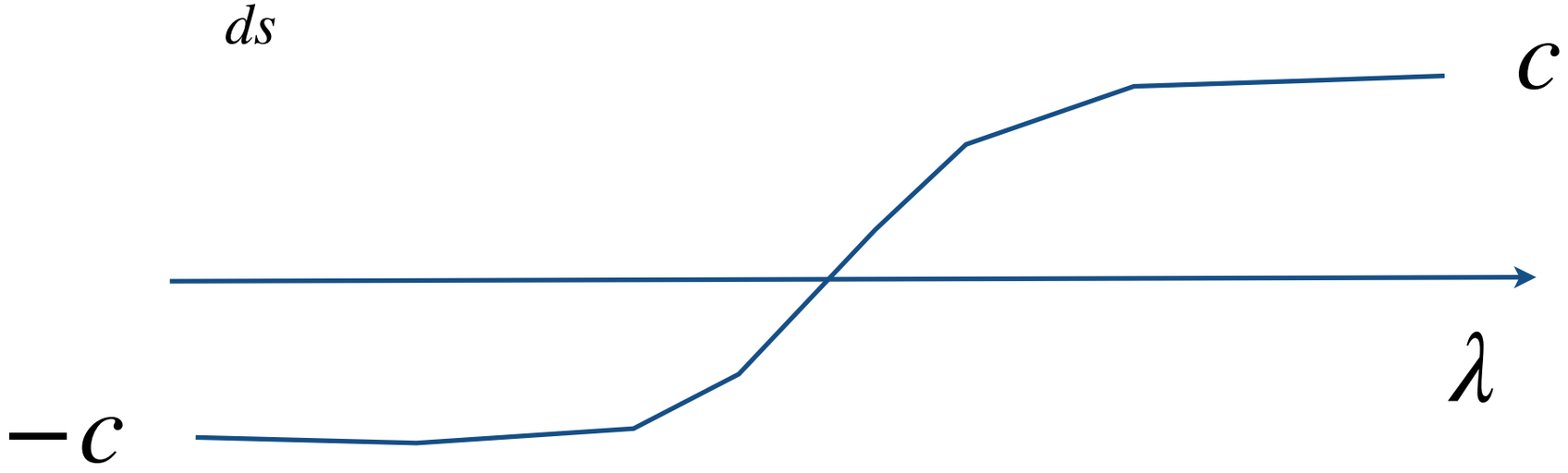
$$u^t = \cosh\lambda, \quad u^x = \sin\lambda$$

It identically satisfies $u^t u^t - u^x u^x = 1$ for all λ.

In the spirit of Galileo, the three-velocity becomes

Particle motion is always **inside** a light cone

$$V = \frac{dx}{dt} = \frac{\frac{dx}{ds}}{\frac{dt}{ds}} = \frac{cu^x}{u^t} : \quad V = c\tanh\lambda \quad \rightarrow |V| < c$$



Summary

- Spacetime is inherently relativistic, as follows from Maxwell's theory of electromagnetic waves and elucidated by Einstein, Minkowski and Lorentz.

- An elegant geometric starting point is given by invariant light cones. All observers see light propagating away in spheres, tracing out light cones in Minkowski spacetime. (Of course, this does not preclude changes in color due to Dopplers shifts.)

- Light cones are generated by the null-rays of photons, whose four-momenta are null-vectors in the Minkowski metric.

- In Minkowski spacetime, four-velocities of massive particles are conveniently expressed by hyperbolic functions as a function of rapidity λ.

Contents - Week 3

W3.1 **Relativistic mechanics**

W3.2 Lorentz transformation

W3.3 Some special relativistic effects

(c)2020 van Putten

Transformation of intervals

Invariance of ds of intervals (dt, dx) and (dt_A, dx_A) measured in the laboratory frame and, respectively, a coming frame of A,

$$ds^2 = (icdt)^2 + dx^2 = (icdt_A)^2 + dx_A^2$$

Look for a unitary transformation on $(icdt, dx)$

Transformation of intervals

Start with the rotation matrix with complex angle $\alpha = e^{i\beta}\lambda$.

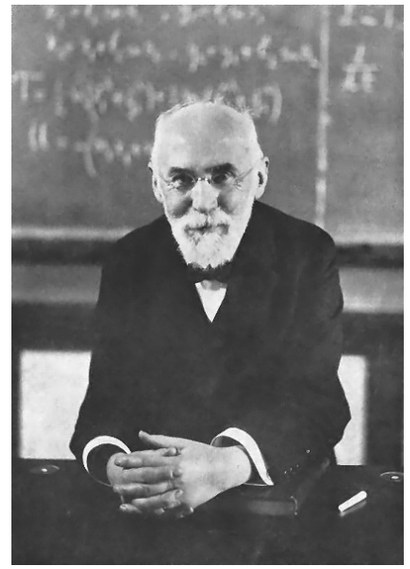
With $\beta = \dfrac{\pi}{2}$, $\alpha = i\lambda$:

$$R(\alpha) = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}, \quad \boxed{\left|R(\alpha)\right| \equiv 1 \;\; (\alpha \; \epsilon \; \mathbb{C}) \quad \text{(unitarity)}}$$

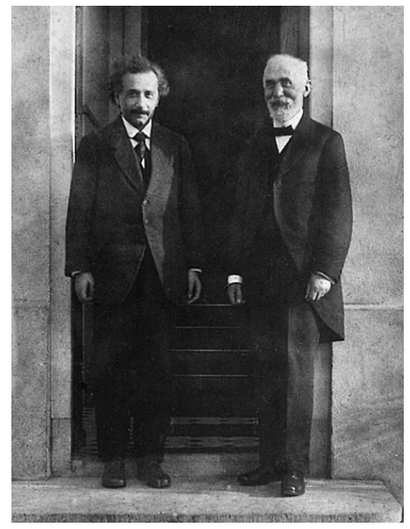
$$R(i\lambda) = \begin{pmatrix} \cosh\lambda & i\sinh\lambda \\ -i\sinh\lambda & \cosh\lambda \end{pmatrix}, \quad R(i\lambda) = \begin{pmatrix} -i & 0 \\ 0 & 1 \end{pmatrix} \Lambda(\lambda) \begin{pmatrix} i & 0 \\ 0 & 1 \end{pmatrix}$$

Similarity transformation preserves unitarity: $\left|\Lambda\right| \equiv 1$

Lorentz transformation: $\Lambda(\lambda) = \begin{pmatrix} \cosh\lambda & \sinh\lambda \\ \sinh\lambda & \cosh\lambda \end{pmatrix}$



H.A. Lorentz (1853-1928)



Transformation of velocities

Apply the rotation with imaginary angle:

$$\begin{pmatrix} -idt \\ dx \end{pmatrix} = R(i\lambda)\begin{pmatrix} -cidt_A \\ dx_A \end{pmatrix} = \begin{pmatrix} -i & 0 \\ 0 & 1 \end{pmatrix}\Lambda\begin{pmatrix} i & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} -idt_A \\ dx_A \end{pmatrix}$$

Lorentz transformation of a spacetime interval:

$$\begin{pmatrix} cdt \\ dx \end{pmatrix} = \Lambda\begin{pmatrix} cdt_A \\ dx_A \end{pmatrix}$$

Recall that $R(\alpha)R(-\alpha) = I :\Rightarrow \Lambda(\lambda)\Lambda(-\lambda) = I$, so that the inverse of a Lorentz boost obtains by changing sign of its rapidity.

Transformation of velocities

Putting $U = \dfrac{dx_A}{cdt_A}$, $V = \tanh\lambda$,

we arrive at the relativistic transformation of velocities

$$W = \frac{dx}{cdt} = \frac{\sinh\lambda cdt_A + \cosh\lambda dx_A}{cdt_A\cosh\lambda + \sinh\lambda dx_A} = \frac{\sinh\lambda + \cosh\lambda U}{\cosh\lambda + \sinh\lambda U}$$

$$W = \frac{U+V}{1+UV}$$

(c)2020 van Putten

Two limiting cases

- Consider an observer *A* on a bicycle watching a bird flying over. *A* measures the bird's velocity (relative to *A*) to be cV. *A*'s velocity on the road is cU. Evidently, both U and V are small, $U, V << 1$. As expected, for small U and V our velocity transformation formula reduces to Galileo's non-relativistic addition of velocities, showing the bird's velocity to a by-stander on the road to be

$$W = \frac{U+V}{1+UV} \simeq U+V \quad (U, V << 1)$$

- On the other hand, consider the same observer *A* on a bicycle watching a laser beam passing by, perhaps pointing at some distant star. *A* measures the velocity of this light to be cV with $V = 1$. *A*'s velocity on the road is cU. The same relativistic transformation formula now shows the same laser light to a by-stander on the road to have a velocity **also** equal to the velocity of light:

$$W = \frac{U+V}{1+UV} = \frac{U+1}{1+U} \equiv 1$$

Tensors: vectors

- The velocity four-vector $u^b = dx^b/ds$ as we have seen, transforms according to the Lorentz transformation. Thus,

$$u^{b'} = \frac{dx^{b'}}{ds} = A^{b'}_b \frac{dx^b}{ds}$$

in terms of the Jacobian,

$$A^{b'}_b = \frac{dx^{b'}}{dx^b}\,.$$

  - Recall that

$$\sum_{b'} A^b_{b'} A^{b'}_c = \sum_{b'} \frac{dx^{b'}}{dx^c}\frac{dx^b}{dx^{b'}} = \delta^c_b,$$

where δ^c_b denotes the Kronecker symbol. In short, $A^{b'}_b$ and $A^b_{b'}$ are each others inverse.

- Thus, the Lorentz transformation is a special case of a Jacobian for the Minkowski line-element parameterized by rapidity.

- We define u^b more generally to be a vector (or vector field) whenever it transforms according to the Jacobian between two coordinate systems.

Coordinate transformations

A scalar field φ has the property that it is entirely observer invariant, i.e., invariant under coordinate transformations $x^b \rightarrow x^{b'}$. Thus,

$$d\varphi = \partial_b \varphi dx^b = \partial_{b'} \varphi dx^{b'}$$

That is

$$\partial_b \varphi = A_b^{b'} \partial_{b'} \varphi$$

with Jacobian

$$A_b^{b'} = \frac{dx^{b'}}{dx^b}$$

As before, the Lorentz transformation is a special case of a Jacobian for the Minkowski line-element parameterized by rapidity. If $A_b^{b'}$ is a Lorentz transformation corresponding to a boost λ, then $A_{b'}^{b}$ is a Lorentz transformation with boost $-\lambda$.

Tensors: 1-forms

The gradient of a scalar field,

$$\omega_a = \partial_a \varphi$$

is a special case of a 1-form.

We define more generally ω_a to be a 1-form by having one lower index that transforms according to the Jacobian between two coordinate systems.

Vectors and 1-forms (transforming according to the above) are examples of tensors, more abstractly defined on manifolds, here of dimension 4=3+1:
three space coordinates and one time coordinate.

Metric tensor

- We define the metric tensor g_{ab}

$$ds^2 = g_{ab} dx^a dx^b$$

to lower or raise indices of tensors

$$u_a = g_{ab} u^b, \quad u^b = g^{ab} u_a$$

where the latter uses the inverse of the metric.

- A special case is the Minkowski metric

$$\eta_{ab} = \begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Photons

From Maxwell's theory, we learn that photon propagation can be described as an electromagnetic wave.

As such, photons - discrete packages in the particle-wave duality - carry a total phase φ. In the plane-wave approximation for a wave propagating along, say, the x-axis, $\varphi = kx - \omega t$, given an angular frequency ω and wave number k.

Thus, $k_a = \partial_a \varphi = (-\omega, k)$ is the 1-form given by the gradient of φ.

Raising indices gives the associated four-vector $k^b = \eta^{ab} k_a = (c^{-2}\omega, k)$.

Energy-momentum vector

- In vacuum, we have the dispersion relation $\omega = ck$, where c does not depend on frequency.

- In what follows, we shall put $c = 1$. The four-vector describing electromagnetic waves (see example) is seen to be a null-vector:

$$k^2 = \partial^a \varphi \partial_a \varphi = -k^t k^t + k^x k^x = 0.$$

- The Planck relations

$$\epsilon = \hbar\omega,\ p = \hbar k,$$

show that photons carry energy according to their frequency.

- We now recognize that the four-vector is an energy-momentum vector:

$$\hbar k^b = \hbar \begin{pmatrix} -\partial_t \varphi \\ \partial_x \varphi \end{pmatrix} = \begin{pmatrix} \epsilon \\ p \end{pmatrix}$$

Rest mass energy

- Four-momentum: $p^a = mu^a$

  As for photons, its t, x-components describe energy and momentum:

$$(E, P) = mu^a = m(\cosh\lambda, \sinh\lambda) : \boxed{E = \sqrt{m^2 + P^2}}$$

  associated with a three-velocity $V = \tanh\lambda$.

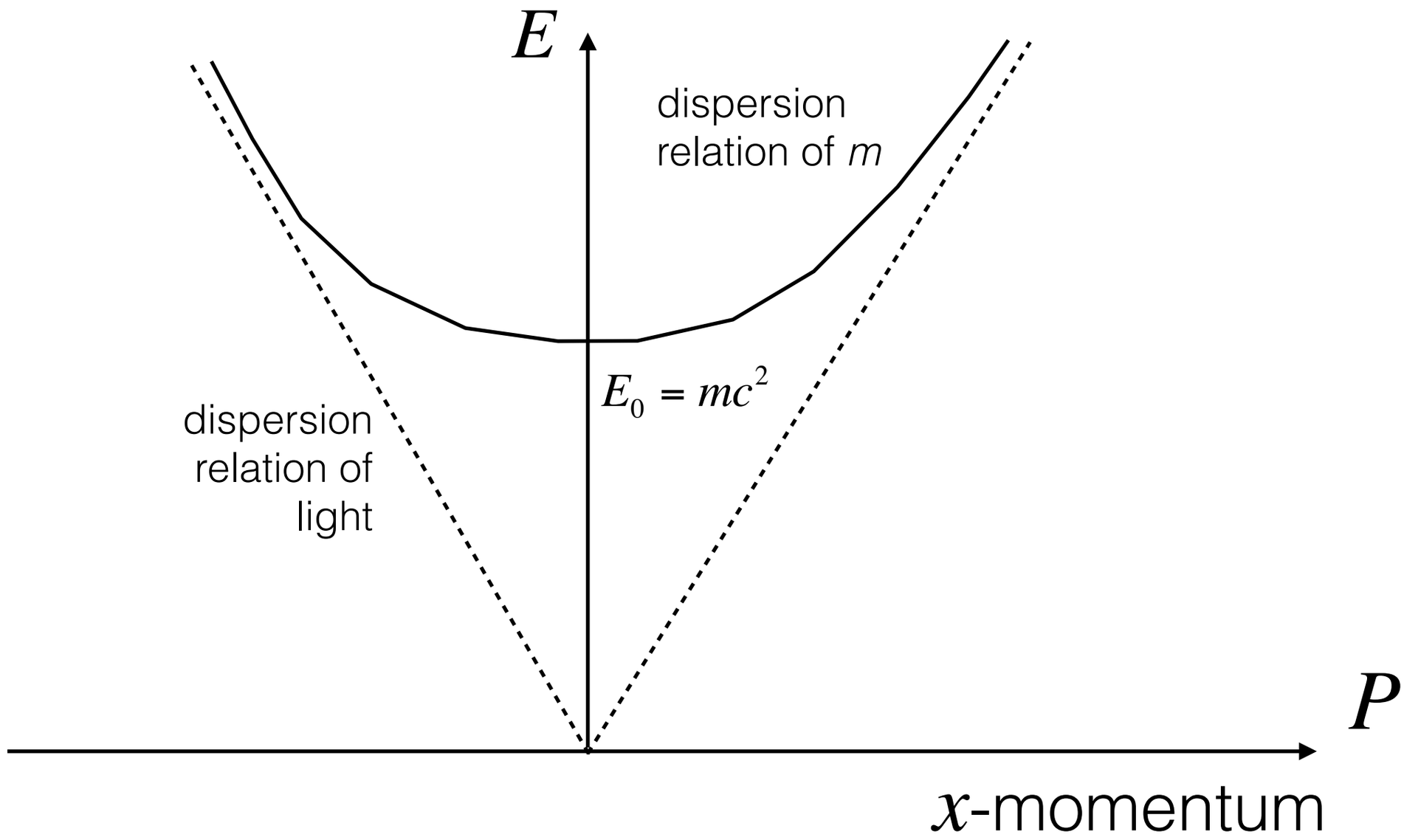
- We recall $\cosh\lambda = \sqrt{1 + \sinh^2\lambda} \cong 1 + \frac{1}{2}\sinh^2\lambda \cong 1 + \frac{1}{2}\tanh^2\lambda \quad (\lambda << 1)$

- Hence, and restoring the velocity of light c,

$$\boxed{E \simeq mc^2 + \frac{1}{2}mV^2}$$

Relativistic particle energy

Rest mass is the Lorentz invariant $m = \sqrt{-p^a p_a}$



Example 1. *Rest mass energy*

Consider a new battery powered by converting 1 g of mass into electrical power at 100% efficiency. (There is yet an X-Prize to be opened for this invention.)

A typical laptop draws a power of about P=100 W, i.e., 100 Joules per second.

For how many years can this battery power the laptop?

Answer:

- The total energy output of the battery over its lifetime will be $E = mc^2 = 1 \times (3 \times 10)^2 \text{g cm}^2\text{s}^{-2} = 9 \times 10^{20} \text{erg}$
- Converting to MKS, $E = 9 \times 10^{13} \text{J}$.
- It can power the laptop for a duration $T = \frac{E}{P} = 9 \times 10^{11} \text{s}$.
- Converted to years, $T \simeq 9 \times 10^{11} / 3 \times 10^{7} \text{yr} = 30{,}000 \text{ yr}$

(c)2020 van Putten

Summary

- Energy and momentum as seen by different observers now transform jointly according to the Lorentz transformation.

- Energy is the time-component of a Lorentz invariant four-vector. This applies universally, for photons (massless) and particles with finite mass.

- While the minimum energy of a photon is zero (at zero frequency), the minimum energy of a massive particle is bounded below by its rest mass times the velocity of light squared. The rest mass of a particle is a Lorentz invariant.

Contents - Week 3

W3.1 **Relativistic mechanics**

W3.2 Lorentz transformation

W3.3 Some special relativistic effects

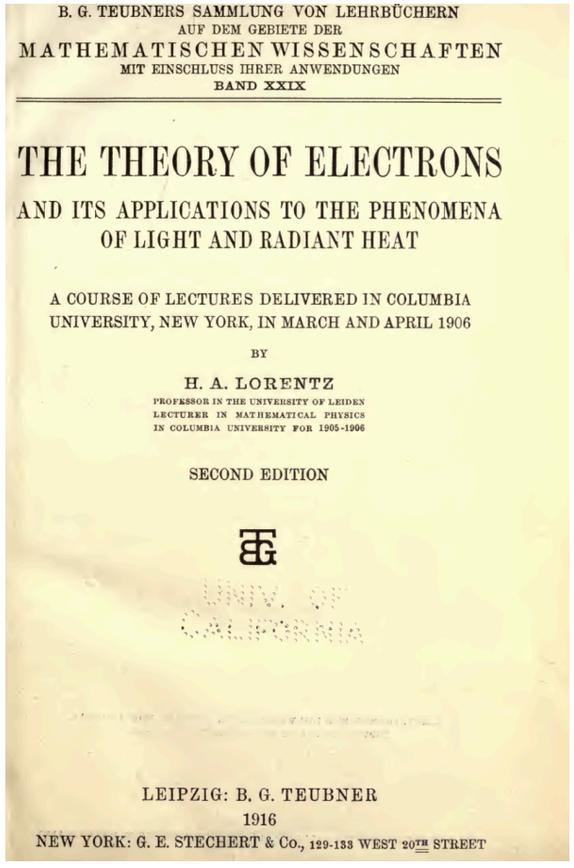
(c)2020 van Putten

Inertial observers

We have looked at Minkowski spacetime, seen by inertial observers at zero acceleration. Lorentz transformations (parameterized by a boost) map Minkowski spacetimes onto itself, leaving light cones invariant.

Upon considering maps modulo Lorentz transformations, all inertial observers are equivalent.

> “It will be clear by what has been said that the impressions received by the two observers A0 and A would be alike in all respects. It would be impossible to decide which of them moves or stands still… nor for saying that either of them is in possession of the "true" times or the "true" lengths.”
>
> H.A. Lorentz (1916)


B. G. TEUBNERS SAMMLUNG VON LEHRBÜCHERN
AUF DEM GEBIETE DER
MATHEMATISCHEN WISSENSCHAFTEN
MIT EINSCHLUSS IHRER ANWENDUNGEN
BAND XXIX

THE THEORY OF ELECTRONS
AND ITS APPLICATIONS TO THE PHENOMENA
OF LIGHT AND RADIANT HEAT

A COURSE OF LECTURES DELIVERED IN COLUMBIA
UNIVERSITY, NEW YORK, IN MARCH AND APRIL 1906

BY

H. A. LORENTZ
PROFESSOR IN THE UNIVERSITY OF LEIDEN
LECTURER IN MATHEMATICAL PHYSICS
IN COLUMBIA UNIVERSITY FOR 1905-1906

SECOND EDITION

UNIV. OF CALIFORNIA

LEIPZIG: B. G. TEUBNER
1916
NEW YORK: G. E. STECHERT & Co., 129-133 WEST 20TH STREET

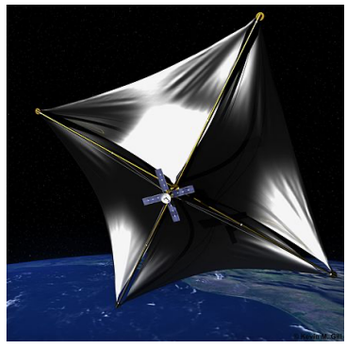
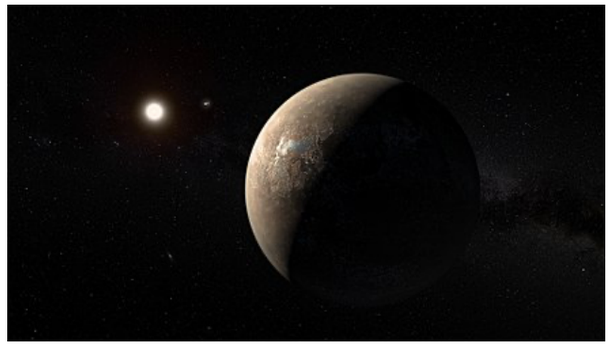
Inertial observers

Even so, the effects of Lorentz transformations can be quite pronounced.

Some special relativistic effects are: Lorentz time-dilation, the apparent shortening of moving objects, Feynman clocks, relativistic Doppler shifts, and apparent superluminal motion.

These topics are now very opportune in our observations of the Extreme Transient Events, e.g., gamma-ray bursts, (micro-)quasars and, to a lesser extent, core-collapse supernovae.

Example 2a. Time dilation

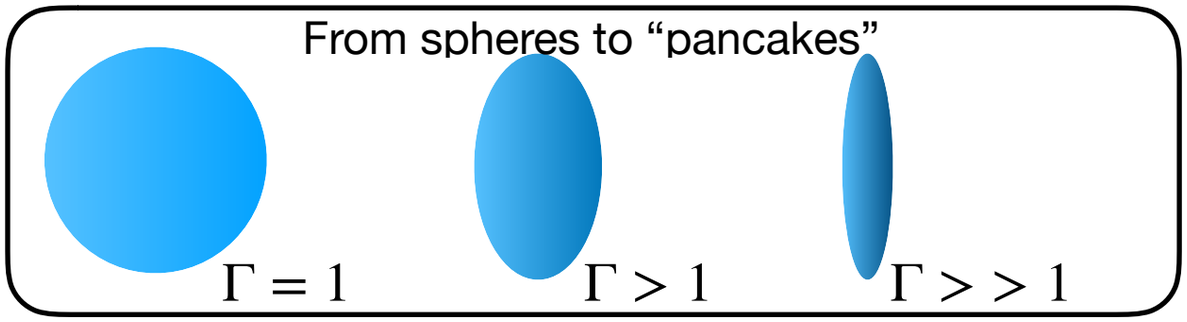
Artist impressions of the Starshot sail and Proxima Centauri B

The “Breakthrough Starshot” initiative aims at sending a micro-satellite to alpha Centauri, the most nearby star (other than the Sun) at a distance of a 1.3pc with exoplanet Proxima Centauri B in its habitable zone. It is designed to be powered by a beam of laser light coming from a source on Earth, propelling it to a velocity $V \simeq 0.1c$, it’s travel time, in Earth’s frame of reference, will be about a decade. What will be the travel time for the on-board clock?

Answer:

- Putting $c = 1$, the velocity four-vector of the micro-satellite $u^b = (\cosh\lambda, \sinh\lambda)$ is a tensor that transforms between different frames of reference according to the Lorentz transformation: $u^b = \Lambda^b_{a'} u^{a'}$. In the comoving frame of reference, we have $u^{b'} = (1,0)$.
- Explicit evaluation gives $\Delta t = \Gamma \Delta t'$, where $\Delta t = d/V \simeq$ 10 yr refers to Earth’s time and $\Delta t'$ refers to time on-broad the satellite. With $1/\Gamma = \sqrt{1-(V/c)^2} \simeq 1 - \frac{1}{2}(V/c)^2 = 0.995$, $\Delta t'$ will one about 0.5% less than Δt, or about two weeks.

Example 2b. Lorentz contraction



Consider the micro-satellite in the shape of a sphere with radius R. It will appear flattened along the direction of motion as it develops a relativistic velocity $V = \tanh \lambda$. Compute the width along the velocity of motion.

Comment: Space-like hyper-surfaces appear tilted by Lorentz boosts

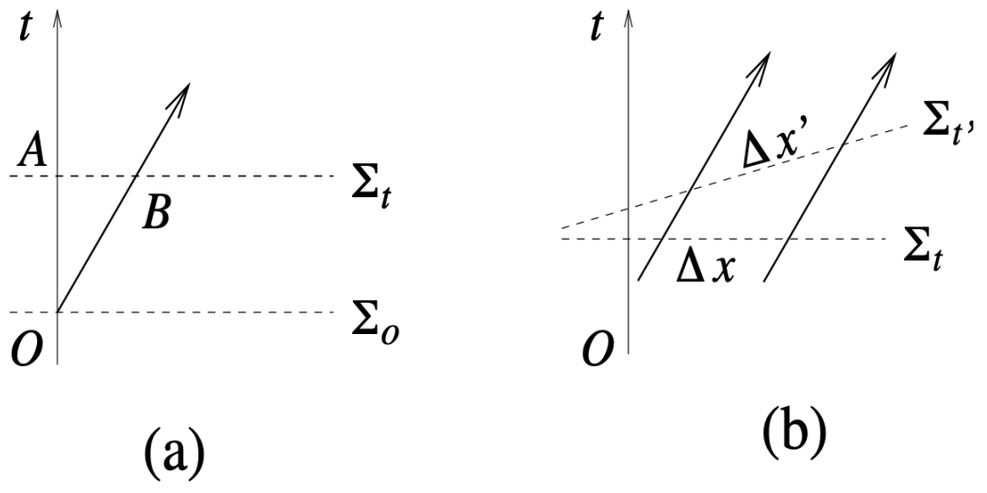
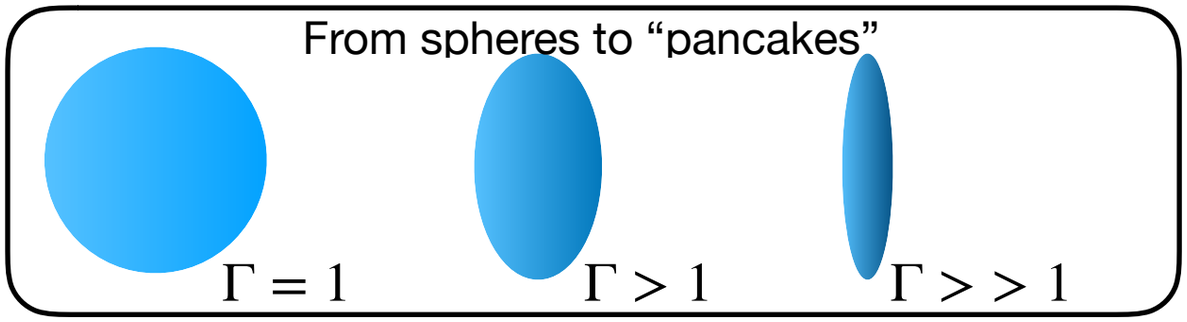


Figure 1.2: (a) Time-dilation is described by the lapse in eigentime of a moving particle (arrow) between two surfaces of constant time Σ_0 and Σ_t in the laboratory frame K. The distance beteen these to surfaces in K is t, corresponding to O and A. The lapse in eigentime is t/Γ upon intersecting Σ_0 at O and Σ_t at B, where Γ is the Lorentz factor of the particle. Moving clocks hereby run slower. (b) The distance between two parallel world-lines (arrows) is the distance between their points of intersection with surfaces of constant time: Σ_t in K and $\Sigma_{t'}$ in the comoving frame K'. According to the Lorentz transformation, $\Delta x = \Delta x'/\Gamma$, showing that moving objects appear shortened and, in the ultrarelativistic case, become so-called “pancakes.”

Reference: van Putten, 2005, “Gravitational radiation, luminous black holes and gamma-ray burst-supernovae” (Cambridge University Press), Ch. 1

Example 2b. Lorentz contraction



Consider the micro-satellite in the shape of a sphere with radius R. It will appear flattened along the direction of motion as it develops a relativistic velocity $V = \tanh\lambda$. Compute the width along the velocity of motion.

Answer:

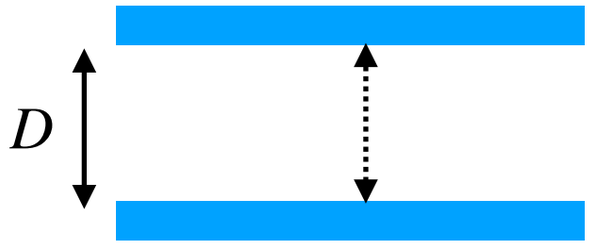
- Let (t', x') denote the comoving frame of reference of the satellite. Its leading and trailing edge of are separated by $2R$. Each has its own world-line. In (t', x'), their space-like separation is given by the spacetime interval $(\Delta t', \Delta x') = (0, 2R)$.

- Let (t, x) denote Earth's lab frame, wherein the width of the satellite is $(\Delta t, \Delta x) = (0, D)$.

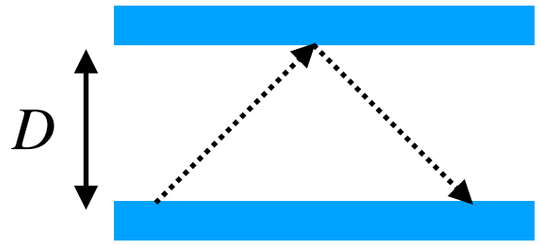
- In general, a Lorentz transformation of $(\Delta t', \Delta x')$ with rapidity λ gives $\Delta t = \Gamma(\Delta t' + V\Delta x')$, $\Delta x = \Gamma(V\Delta t' + \Delta x')$. Measuring a width, we insist $\Delta t = 0$, leaving $\Delta x = \Gamma(1 - V^2)\Delta x'$. Note that $\Delta t' = -V\Delta x'$ is non-zero, by the fact that the x-axis of Earth's frame of reference appears tilted (to time-like) in (t', x'). With $\Delta x' = 2R$, we conclude

$$D = \frac{2R}{\Gamma}\,.$$

Example 2c. Feynman's photon clock



(a) V=0



(b) V>0

R.P. Feynman 1918-1988

Photon bouncing between two mirrors, standing still (a) or moving (b) along the x-axis.

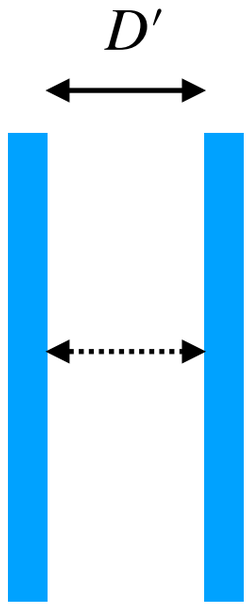
$$(a):\ P' = 2\frac{D}{c},\ f' = \frac{c}{2D}$$

$$(b):\ V = c\sin\alpha:\ W = c\cos\alpha = c\sqrt{1-(V/c)^2} = c/\Gamma,\ P = 2\frac{D}{W},\ f = \frac{c}{2D}\Gamma^{-1}$$

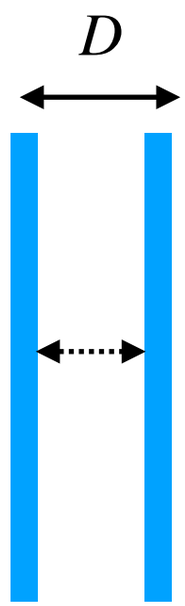
$$\boxed{f = \frac{f'}{\Gamma}}$$

(c)2020 van Putten

.. rotated



(a) V=0



(b) V>0

R.P. Feynman 1918-1988

Photon bouncing between two mirrors, standing still (a) or moving (b) along the x-axis.

$$(a):\ P' = 2\frac{D'}{c},\ f' = \frac{c}{2D'}$$

$$(b):\ \Delta t_{\pm} = D \pm V\Delta t_{\pm}:\ P = \Delta t_{+} + \Delta t_{-} = \frac{2Dc}{c^2 - V^2} = 2Dc\Gamma^2,\ f = \frac{2D}{c}\Gamma^{-2}$$

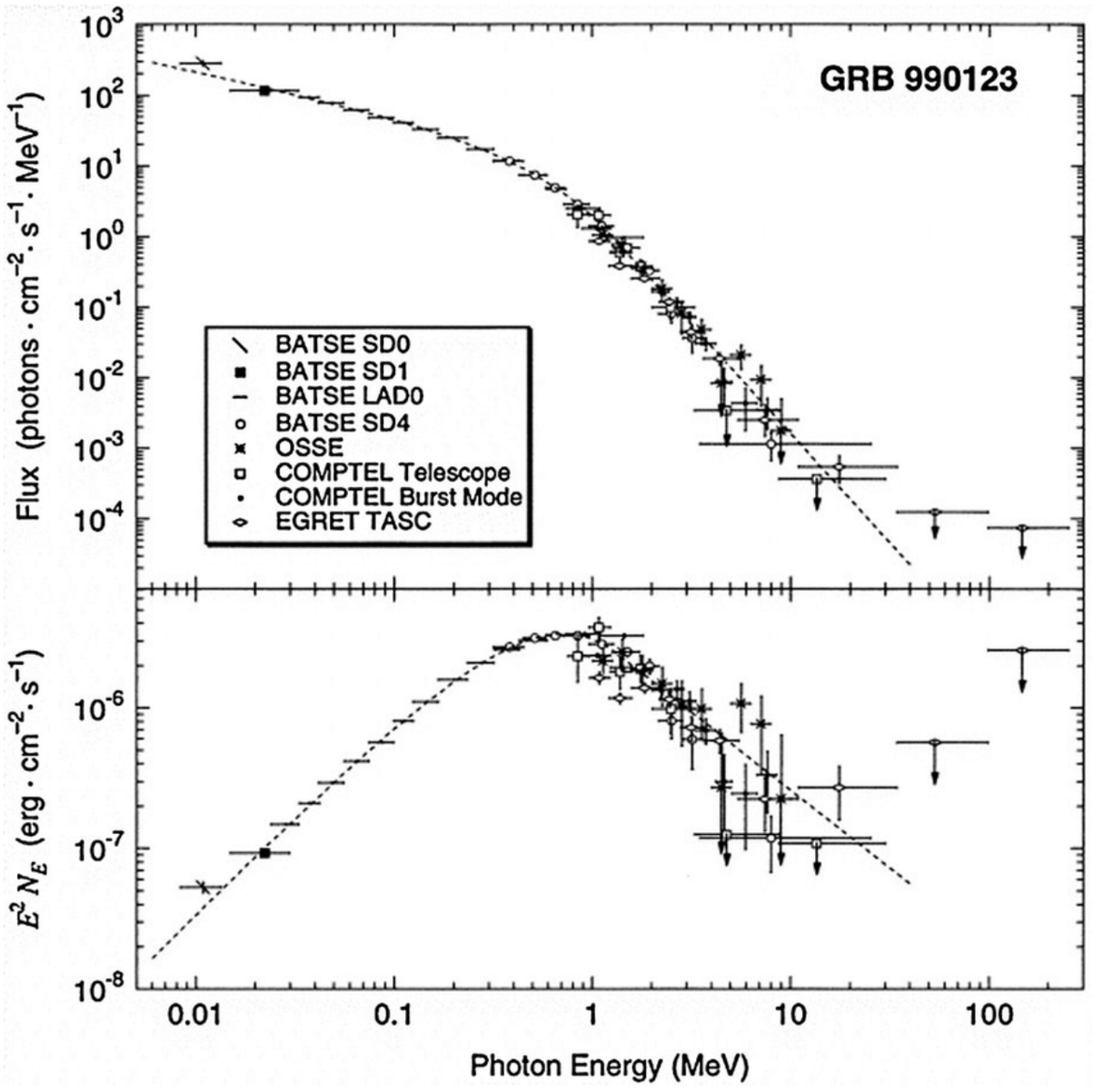
Clock frequency does not depend on orientation, result should be the same as the previous result, when motion is parallel to the mirrors:

$$f = \frac{2D'}{c}\Gamma^{-1} = \frac{2D}{c}\Gamma^{-2}: \boxed{D = \frac{D'}{\Gamma}.}$$

Example 3. *Doppler factor*

A gamma-ray burst (GRB) emits a photon with energy $\epsilon' = 50$ keV in the rest frame of a shock in a relativistic outflow with Lorentz factor $\Gamma = 10 - 100$. Ignoring cosmological redshift, compute the energy seen by one of our GRB-observatories such as *Swift* or GBM-*Fermi.*

Band function in the energy spectrum of GRB 990123 (E_p = 720 keV) in Briggs et al. 1999



M. Briggs, D. Band, R. Kippen, R. Preece, C. Kouveliotou, J. Van Paradijs, G. Share, R. Murphy, S. Matz, A. Connors et al., 1999, ApJ, 524, 82

Willingale, R., Meszaros, P., 2017, Space Science Reviews, 207, 63

Example 3. *Doppler factor*

Answer:

- Putting $c = 1$, photons of energy ϵ' are described by null-vectors $k^{b'} = \epsilon'(1,1)$. The photon appears to any other observer with null-vector k^b following a Lorentz transformation of k^b: $k^b = \Lambda^b_{a'} k^{a'} = \epsilon(1,1)$, again a null-vector by Lorentz invariance of light cones.

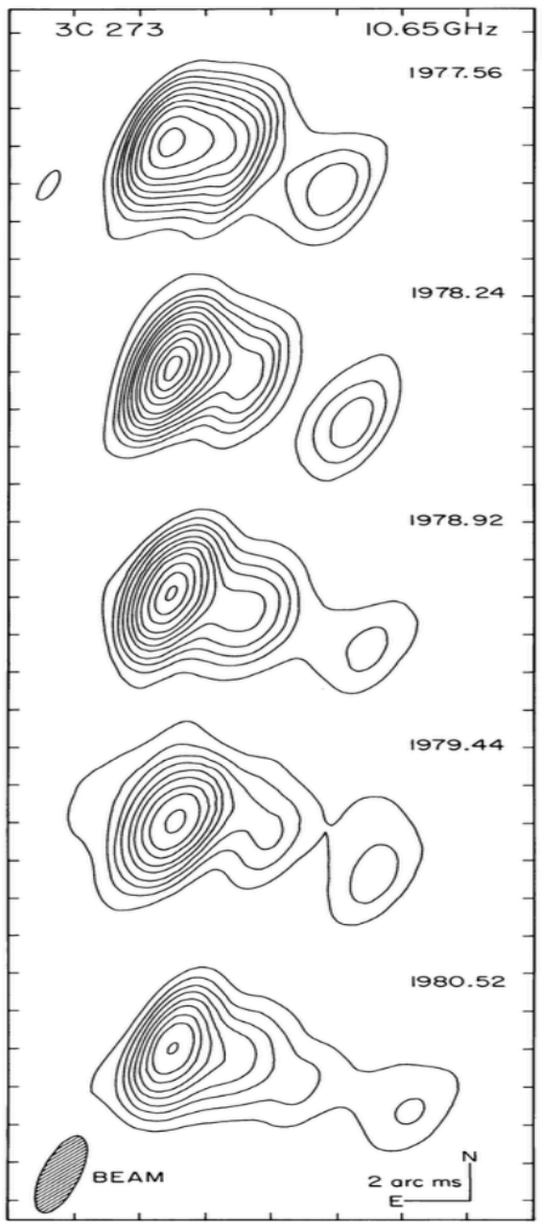
- Explicit evaluation gives $\epsilon = \epsilon' \Gamma(1 + V),\ V = \tanh\lambda$, where $\Gamma = \cosh\lambda$ is the Lorentz factor of the source (relative to the observer). Noting that $\Gamma = 1/\sqrt{1 - V^2}$, a further evaluation gives $\epsilon = \delta\epsilon'$ in terms of the relativistic Doppler factor

$$\delta \equiv \sqrt{\frac{1+V}{1-V}}\,.$$

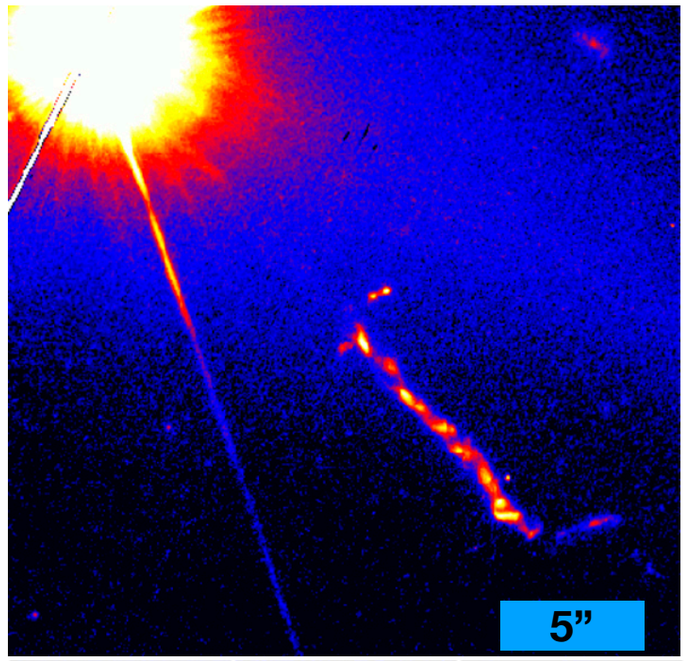
- For large Lorentz factors (velocities close to the velocity of light), we have $\delta \simeq 2\Gamma$. For $\Gamma = 10 - 100$, the observed photon energy, therefore, is $\epsilon = 1 - 10$ MeV.

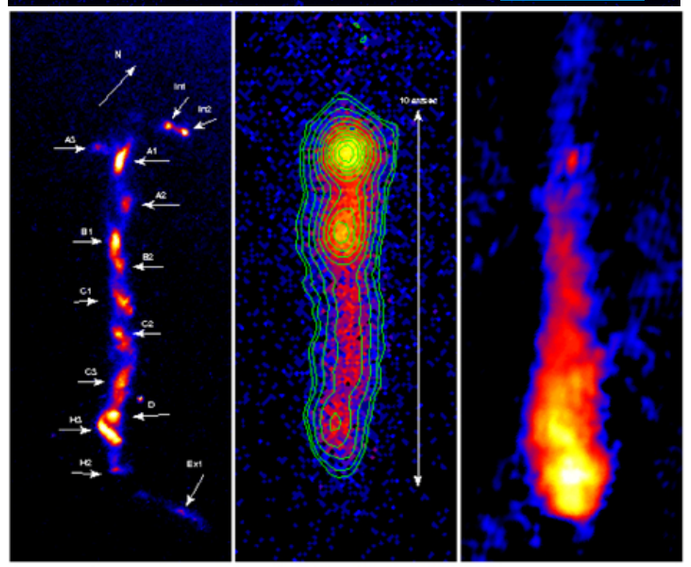
Example 4. Apparent super-luminal motion

3C273 is a quasar at redshift $z = 0.156$ with a powerful radio jet. The jet appears with various bright features emanating from the core, when observed over an extended period of time (years). Viewed as projections on the sky, this transverse motion $V_\perp \simeq 10c$ appears faster than the velocity of light.



A Very Large Baseline Interferometry (VLBI) contour map of five epochs on an ejection event in the quasar 3C273 in the radio (10.65 GHz). (Pearson, T.J., et al., 1981, Nature, 280, 365)





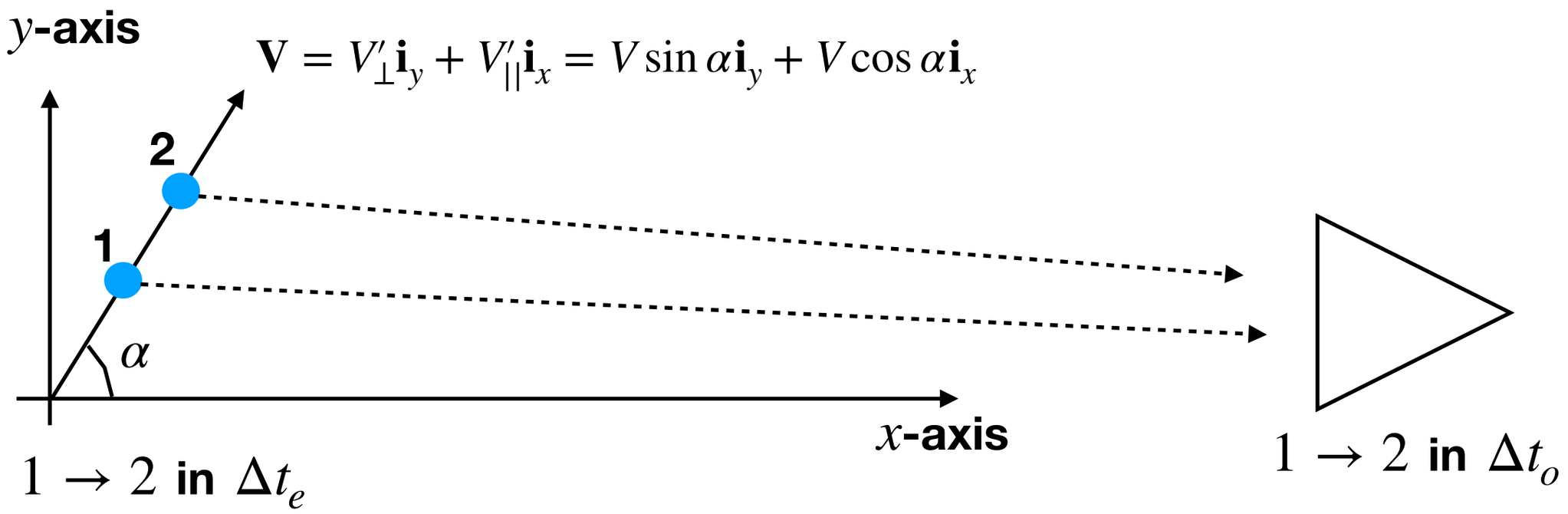
https://chandra.harvard.edu/photo/2000/0131/0131_optical.jpg

(c)2020 van Putten

Example 4. Apparent super-luminal motion

Answer:

Reference: van Putten, 2005, “Gravitational radiation, luminous black holes and gamma-ray burst-supernovae” (Cambridge University Press), Ch. 1



$\beta = V/c$:

$$V_{\perp} = \frac{\Delta y}{\Delta t_o} = \frac{V\sin\alpha\Delta t_e}{\Delta t_e - (V\cos\alpha\Delta t_e)/c} = c\frac{\beta\sin\alpha}{1-\beta\cos\alpha} = f_{\beta}(\alpha)$$

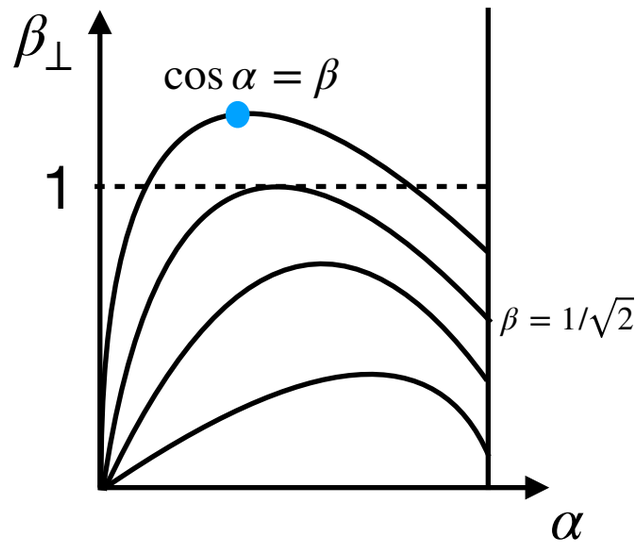
(c)2020 van Putten

Example 4. Apparent super-luminal motion

Answer:

$$\beta = V/c: \quad V_\perp = \frac{\Delta y}{\Delta t_o} = \frac{V \sin\alpha \Delta t_e}{\Delta t_e - (V\cos\alpha\Delta t_e)/c} = c\frac{\beta\sin\alpha}{1-\beta\cos\alpha} = f_\beta(\alpha)$$

$\beta_\perp = V_\perp/c$:



$$\beta_\perp = f_\beta(\alpha) \le \beta\Gamma,\ \Gamma \ge \sqrt{1+\beta_\perp^2}$$

So, $\beta_\perp \simeq 10 \Rightarrow \Gamma \gtrsim 10$**.**

Summary

- Lorentz transformations shows the transformations of spacetime intervals needed to preserve that the velocity of light is universal, emphasized by Einstein as the Principle of Special Relativity.

- Modern observational astronomy show examples abound from the Extreme Transient Universe of relativistic and even ultra-relativistic motions, common in outflows from compact objects.

- These transient sources pose deep questions on the physical mechanism producing (ultra-)relativistic astrophysical outflows. As you may expect, these are now believed to be associated with neutron stars and black holes, though a detailed discussion we leave for another time.

(c)2020 van Putten