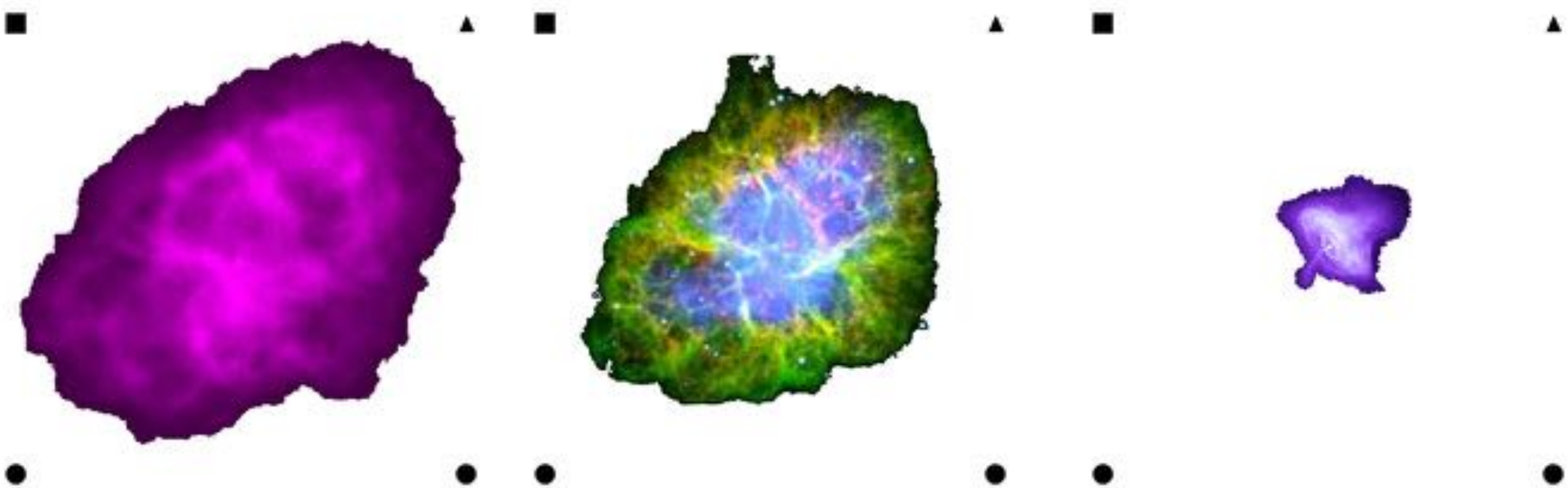
Comments on synchrotron radiation in the Crab

Multiwavelength observations

Crab synchrotron emission (http://chandra.harvard.edu/edu/formal/composites/crab_overlays.html)



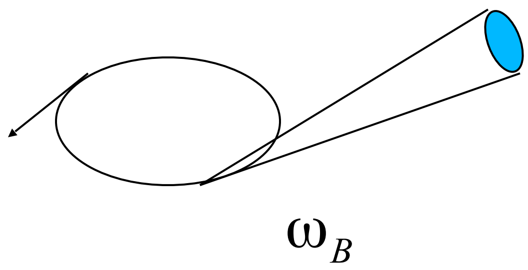
Radio: *extended, low energy electrons*

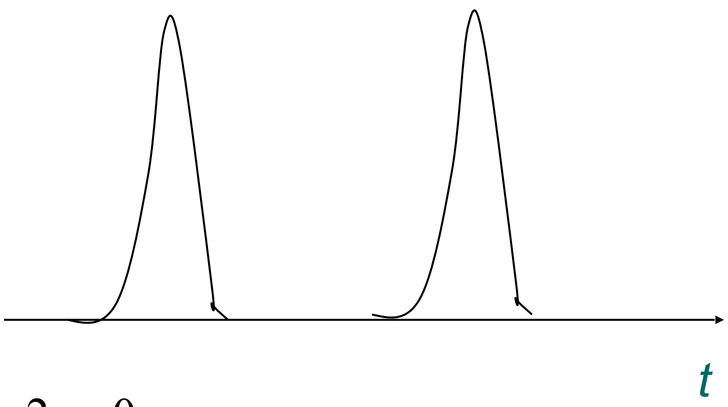
Optical: *intermediate energy electrons*

X-ray: *compact, close to the source, highest energy electrons*

Synchrotron radiation energies

Emission binned in flashes of light, as conical emission from each electron passes by observer. Characteristic photon frequency scales with square of the Lorentz factor:





$$\omega_c = \frac{3}{2}\gamma^3\omega_B \sin\alpha = \frac{3}{2}\gamma^2\omega_B^0 \sin\alpha$$

$$\omega_B \equiv \frac{eB}{\gamma mc} \qquad \textit{gyration frequency}$$

$$\omega_B^0 \equiv \frac{eB}{mc} \qquad \textit{cyclotron frequency}$$

Synchrotron radiation spectra

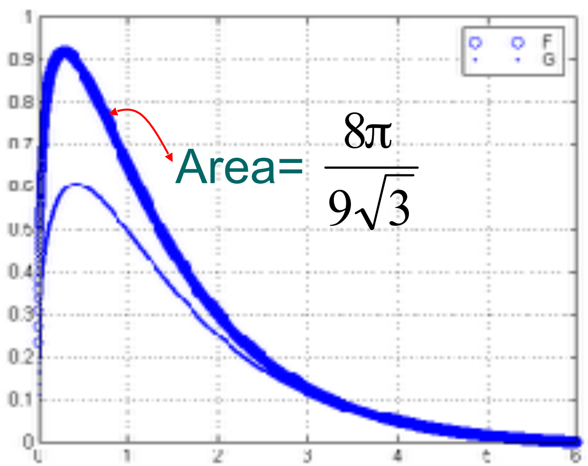
Below a critical frequency: spectrum is optically thick (black body)

Above: optically thin (power law from power law energy distribution of electrons)

$$F(x) \sim \frac{8\pi}{9\sqrt{3}}\delta(x-1)$$

$$\omega_c = \frac{3}{2}\gamma^3\omega_B \sin\alpha \qquad \text{(A)}$$

F, G

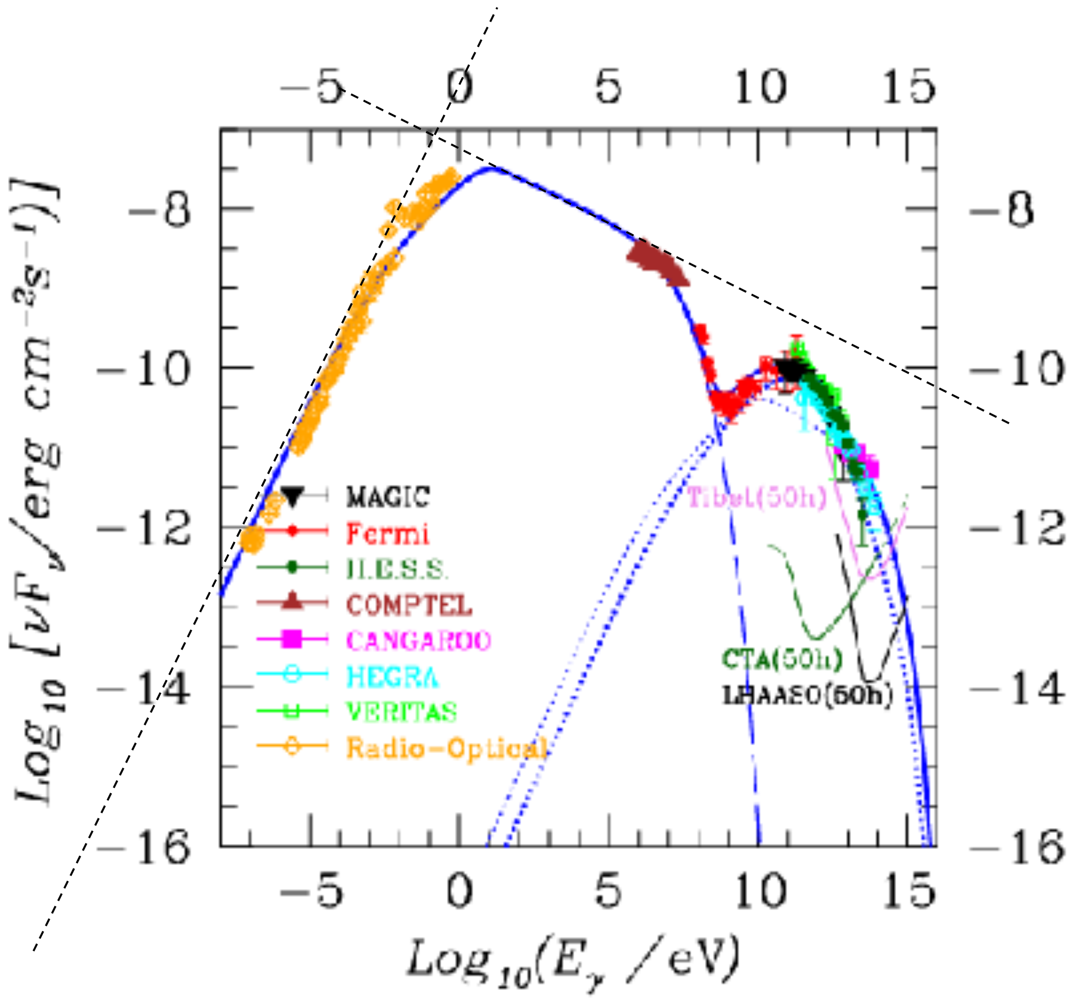


$$\omega_c = 0.29\times\frac{3}{2}\gamma^3\omega_B \sin\alpha \qquad \text{(B)}$$

We commonly use approximation (A)

Synchrotron radiation spectra

Kohri, Yutaka & Kunihito
MNRAS, 424, 2249
Manchester, Staveley-Smith &
Kesteven, ApJ, 411, 756



break between radio and high energy-energy spectra

$$\nu_b \approx 10^{14}\ \mathrm{Hz}$$

Manchester et al.

$$(\nu_b \approx 2\times 10^{13}\ \mathrm{Hz})$$

------------------------ *extrapolations of radio and high-energy spectra*

Putting it all together:

$$u_B = \frac{1}{8\pi} B^2$$

$$< P >_\alpha = \frac{4}{3}\left(\frac{m_e}{m}\right)^2 \sigma_T c \gamma^2 u_B$$

$$t_{syn} = \frac{\gamma m c^2}{< P >_\alpha} = \frac{6\pi m_e c^2}{\sigma_T c}\left(\frac{m}{m_e}\right)^3 \frac{1}{\gamma B^2} \cong 10^9 \left(\frac{m}{m_e}\right)^3 \gamma^{-1} B^{-2}$$

$$\nu_c = \frac{3}{2}\gamma^2 \frac{eB}{mc}: \quad \gamma = \sqrt{\frac{2mc}{3eB}}\nu_c^{\frac{1}{2}}$$

$$\Rightarrow \quad t_{syn} \cong 5\times 10^{11} B^{-\frac{3}{2}} \nu_c^{-\frac{1}{2}} \quad (m = m_e)$$

Results for the Crab:

$$\nu_c \approx 0.2 - 1 \times 10^{14}\ \mathrm{Hz}:$$

$$t_{syn} \cong 5\times 10^{11} B^{-\frac{3}{2}} \nu_c^{-\frac{1}{2}}: \quad B \cong \left(\frac{5\times 10^{11}}{t_{syn}\nu_c^{\frac{1}{2}}} \right)^{\frac{2}{3}} \cong 0.2\,\mathrm{mG}$$

$$\omega_B^0 = \frac{eBc}{m_e c^2} \cong 3\,\mathrm{kHz}$$

$$\gamma = \sqrt{\frac{2\nu_c}{3\omega_B^0}} \cong 10^5$$